

Practice Problems 1

Graph each system as a pair of lines in the xy -plane, solve and interpret the answer.

$$1) \quad \begin{aligned} 2x + y &= 4 \\ x - y &= 2 \end{aligned}$$

$$2) \quad \begin{aligned} x + 3y &= 2 \\ -x + 2y &= 3 \end{aligned}$$

$$3) \quad \begin{aligned} x - y &= 1 \\ -2x + 2y &= 5 \end{aligned}$$

$$4) \quad \begin{aligned} \frac{x+3}{4} + \frac{y-1}{3} &= 1 \\ 2x - y &= 12 \end{aligned}$$

Solve the following systems represented by the augmented matrices using Gaussian elimination and back substitution.

$$5) \quad \left(\begin{array}{ccc|c} 1 & -1 & 0 & 3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{array} \right)$$

$$6) \quad \left(\begin{array}{ccc|c} 2 & 1 & -1 & 3 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{array} \right)$$

$$7) \quad \left(\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 1 & -2 & 1 & -2 \\ 1 & 0 & 1 & 0 \end{array} \right)$$

Solve the following systems by writing them in matrix form and using Gauss-Jordan elimination.

$$8) \quad \begin{aligned} x + 2y &= 7 \\ 2x + y &= 8 \end{aligned}$$

$$9) \quad \begin{aligned} -3x + 5y &= -22 \\ 3x + 4y &= 4 \\ 4x - 8y &= 32 \end{aligned}$$

$$10) \quad \begin{aligned} x_1 - 3x_3 &= -2 \\ 3x_1 + x_2 - 2x_3 &= 5 \\ 2x_1 + 2x_2 + x_3 &= 4 \end{aligned}$$

$$11) \quad \begin{aligned} x_1 + x_2 - 5x_3 &= 3 \\ x_1 - 2x_3 &= 1 \\ 2x_1 - x_2 - x_3 &= 0 \end{aligned}$$

12) $3x + 3y + 12z = 6$
 $x + y + 4z = 2$
 $2x + 5y + 20z = 10$
 $-x + 2y + 8z = 4$

13) Find the value of k such that the system represented by the following augmented matrix is consistent.

$$\left(\begin{array}{cc|c} 2 & -1 & 3 \\ -4 & 2 & k \\ 4 & -2 & 6 \end{array} \right)$$

14) Determine whether the following matrices are elementary or not. If they are state the elementary row operation used to produce it.

a) $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

15) Factor the following matrices into a product of elementary matrices.

a) $\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 4 & -1 \\ 3 & -1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

16) Find the inverses of the following matrices by adjoining the identity matrix then performing Gauss-Jordan elimination.

a) $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ 7 & 16 & -21 \end{pmatrix}$

- 17) Find x such that the following matrix is singular (has no inverse).

$$\begin{pmatrix} 3 & x \\ -2 & -3 \end{pmatrix}$$

- 18) Find the LU -factorisation of the following matrix.

a) $\begin{pmatrix} -2 & 1 \\ -6 & 4 \end{pmatrix}$

b) $\begin{pmatrix} 3 & 0 & 1 \\ 6 & 1 & 1 \\ -3 & 1 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 10 & 12 & 3 \end{pmatrix}$

- 19) Solve the following system by finding the LU -factorisation of the coefficient matrix then solving the lower and upper triangular systems.

$$\begin{aligned} 2x + y &= 1 \\ y - z &= 2 \\ -2x + y + z &= -2 \end{aligned}$$