Practice Problems 2

1) State the additive identity of the following vector spaces.
a) $\mathbb{R}^4$ (the set of real vectors in 4 dimensions).
b) $C(-\infty, \infty)$ (the set of continuous functions on the real line).
c) M_{23} (the set of 2×3 matrices).
d) P_3 (the set of 3rd degree or less polynomials).

2) State whether the following sets with with the standard operations form a vectors space or not. If they are not a vector space then identify at least one of the axioms that fails.
a) M_{46}.
b) The set of all 3rd degree polynomials, P_3^*.
c) The set of all 1st degree polynomials, $ax + b$, with $a \neq 0$, whose graphs pass through the origin.
d) The set $\{(x, y): x \geq 0, y$ is a real number$\}$.
e) The set $\{(x, y): x \geq 0, y \geq 0\}$.
f) The set $\{\left(x, \frac{1}{2}x\right): x$ is a real number$\}$.
g) The set of all 2×2 matrices of the form $\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ with $a, b, c \in \mathbb{R}$.
h) The set of all 2×2 matrices of the form $\begin{pmatrix} a & b \\ c & 1 \end{pmatrix}$ with $a, b, c \in \mathbb{R}$.
i) The set of all 2×2 singular matrices.
j) The set of all 2×2 diagonal matrices.

3) Verify the 10 axioms in detail for M_{22} with the standard operations.

4) Show that the subsets, W, are subspaces of the specified vector spaces, V.
a) $W = \{(x_1, x_2, x_3, 0): x_1, x_2, x_3$ are real numbers. $V = \mathbb{R}^4$.
b) W is the set of all 3×2 matrices of the form $\begin{pmatrix} a & b \\ a+b & 0 \\ 0 & c \end{pmatrix}$ where $a, b, c \in \mathbb{R}$. $V = M_{32}$.

5) Show that the subsets, W, are not subspaces.
a) W is the set of all vectors in $\mathbb{R}^3$ whose third component is -1.
b) W is the set of all vectors in $\mathbb{R}^2$ whose components are rational numbers.
c) W is the set of all matrices with zero determinants.

6) Determine whether the subsets, W, are subspaces of $\mathbb{R}^3$ or not.
a) $W = \{(a, b, a + 2b): a, b$ are real numbers$\}$.
b) $W = \{(x_1, x_2, x_1x_2): x_1, x_2$ are real numbers$\}$.

7) Construct a geometric figure that illustrates why a line in $\mathbb{R}^2$ that does not pass through the origin is not closed under vector addition.

8) Let W be the set of vectors of the form $\begin{pmatrix} 2b+3c \\ -b \\ 2c \end{pmatrix}$. Find vectors, $\mathbf{u}$ and $\mathbf{v}$, such that $W = Span\{\mathbf{u}, \mathbf{v}\}$ and hence that W is a subspace of $\mathbb{R}^3$.

9) Either find a set of vectors that span W or show that it is not a subspace.

a) W is the set of vectors of the form $\begin{pmatrix} 1 \\ 3a-5b \\ 3b+2a \end{pmatrix}$.

b) W is the set of vectors of the form $\begin{pmatrix} 4a+3b \\ 0 \\ a+3b+c \\ 3b-2c \end{pmatrix}$.

10) Determine whether each vector can be written as a linear combination of the vectors in S.

a) $S = \{(2,-1,3), (5,0,4)\}$

$\mathbf{u} = (1,1,-1)$, $\mathbf{v} = (8,-1/4,27/4)$, $\mathbf{w} = (1,-8,12)$.

b) $S = \{(2,0,7), (2,4,5), (2,-12,13)\}$

$\mathbf{u} = (-1,5,-6)$, $\mathbf{v} = (-3,15,18)$, $\mathbf{w} = (1/3,4/3,1/2)$.

11) Determine whether the set, S, spans $\mathbb{R}^2$. If it does not, give a geometric description of the subspace that it does span.

a) $S = \{(2,1), (-1,2)\}$.

b) $S = \{(-3,5)\}$.

c) $S = \{(1,3), (-2,-6), (4,12)\}$.

d) $S = \{(-1,4), (4,-1), (1,1)\}$.

12) Determine whether the set, S, spans $\mathbb{R}^3$. If it does not, give a geometric description of the subspace that it does span.

a) $S = \{(4,7,3), (-1,2,6), (2,-3,5)\}$.

b) $S = \{(-2,5,0), (4,6,3)\}$.

c) $S = \{(1,-2,0), (0,0,1), (-1,2,0)\}$.

13) Determine whether the set, S, is linearly independent or linearly dependent.

a) $S = \{(-2,2), (3,5)\}$.

b) $S = \{(0,0), (1,-2)\}$.

c) $S = \{(-4,-3,4), (1,-2,3), (6,0,0)\}$.

d) $S = \{(4,-3,6,2), (1,8,3,1), (3,-2,-1,0)\}$.

e) $S = \{(0,0,01), (0,0,1,1), (0,1,1,1), (1,1,1,1)\}$.

14) Show that the following sets are linearly dependent then express one of the vectors in the set as a linear combination of the others.

a) $S = \{(3,4), (-1,1), (2,0)\}$.

b) $S = \{(1,1,1), (1,1,0), (0,0,1)\}$.

15) Determine whether S is a basis for the indicated vector space.

a) $S = \{(3,-2),(4,5)\}$ for $\mathbb{R}^2$.

b) $S = \{(1,5,3),(0,1,2),(0,0,6)\}$ for $\mathbb{R}^3$.

c) $S = \{(0,3,-2),(4,0,3),(-8,15,-16)\}$ for $\mathbb{R}^3$.

d) $S = \{(-1,2,0,0),(2,0,-1,0),(3,0,0,4),(0,0,5,0)\}$ for $\mathbb{R}^4$.

e) $S = \left\{\begin{pmatrix}2 & 0\\0 & 3\end{pmatrix},\begin{pmatrix}1 & 4\\0 & 1\end{pmatrix},\begin{pmatrix}0 & 1\\3 & 2\end{pmatrix},\begin{pmatrix}0 & 1\\2 & 0\end{pmatrix}\right\}$ for M_{22}.

16) Determine whether S is a basis for $\mathbb{R}^3$. If it is then write $\mathbf{u} = (8,3,8)$ as a linear combination of the vectors in S.

a) $S = \{(4,3,2),(0,3,2),(0,0,2)\}$.

b) $S = \{(0,0,0),(1,3,4),(6,1,-2)\}$.

17) Is $\mathbf{w} = (1,-1,1)^{\mathrm{T}}$ in Nul $\mathbf{A}$?

$$\mathbf{A} = \begin{pmatrix}2 & 6 & 4\\-3 & 2 & 5\\-5 & -4 & 1\end{pmatrix}$$

18) Determine if $\mathbf{b}$ is in the column space of $\mathbf{A}$ or not?

a) $\mathbf{A} = \begin{pmatrix}-1 & 2\\4 & 0\end{pmatrix}$, $\mathbf{b} = (3,4)^{\mathrm{T}}$.

b) $\mathbf{A} = \begin{pmatrix}1 & 3 & 2\\-1 & 1 & 2\\0 & 1 & 1\end{pmatrix}$, $\mathbf{b} = (1,1,0)^{\mathrm{T}}$.