Practice Problems 5

1) Determine whether the following sets are orthogonal, orthonormal or neither.
a) $\{(2,-4),(2,1)\}$
b) $\{(-4,6),(5,0)\}$
c) $\left\{\left(\frac{\sqrt{2}}{2},0,\frac{\sqrt{2}}{2}\right),\left(-\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6}\right),\left(\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3},-\frac{\sqrt{3}}{3}\right)\right\}$

2) Find the coordinates of $\mathbf{x}$ (given in the standard basis for $\mathbb{R}^n$) relative to the orthonormal basis, B.
a) $\mathbf{x}=(1,2)$, $\left\{\left(-\frac{2\sqrt{13}}{13},\frac{3\sqrt{13}}{13}\right),\left(\frac{3\sqrt{13}}{13},\frac{2\sqrt{13}}{13}\right)\right\}$
b) $\mathbf{x}=(2,-2,1)$, $\left\{\left(\frac{\sqrt{10}}{10},0,\frac{3\sqrt{10}}{10}\right),(0,1,0),\left(-\frac{3\sqrt{10}}{10},0,\frac{\sqrt{10}}{10}\right)\right\}$

3) Use Gram-Schmidt orthonormalisation to transform the basis in an orthonormal one using the Eulcidean inner product (normal dot product).
a) $B=\{(3,4),(1,0)\}$
b) $B=\{(4,-3,0),(1,2,0),(0,0,4)\}$
c) $B=\{(3,4,0,0),(-1,1,0,0),(2,1,0,-1),(0,1,1,0)\}$

4) Find an orthonormal basis for the solution space of the following homogeneous system.

$$\begin{aligned}2x_1+x_2-6x_3+2x_4&=0\\x_1+2x_2-3x_3+4x_4&=0\\x_1+x_2-3x_3+2x_4&=0\end{aligned}$$

5) Transform the basis $B=\{(2,-1),(-2,10)\}$ into an orthonormal basis using the inner product,

$$\langle\mathbf{u},\mathbf{v}\rangle=2u_1v_1+u_2v_2$$

6) Find bases for the four fundamental subspaces of the following matrix.

$$\mathbf{A}=\begin{pmatrix}1&1&-1\\0&2&1\\1&3&0\end{pmatrix}$$

7) Are the following sets of vectors orthogonal?
a) $S_1=Span\{(2,1,-1),(0,1,1)\}$, $S_2=Span\{(-1,2,0)\}$
b) $S_1=Span\{(1,1,1,1)\}$, $S_2=Span\{(-1,1,-1,1),(0,2,-2,0)\}$

8) Find the orthogonal complement of S.
a) $S=Span\{(1,2,0,0),(0,1,0,1)\}$
b) S is the subspace of $\mathbb{R}^3$ consisting of the xz-plane.

9) Find the projection of $\mathbf{v}$ onto the subspace, S.

a) $S = Span\{(0,0,-1,1),(0,1,1,1)\}$, $\mathbf{v} = (1,0,1,1)$

b) $S = Span\{(1,0,1),(0,1,1)\}$, $\mathbf{v} = (2,3,4)$

c) S is the column space of $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$, $\mathbf{v} = (2,-2,1)$

10) Find the least squares solution to the system $\mathbf{Ax} = \mathbf{b}$.

a) $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{pmatrix}$, $\mathbf{b} = (2,0,-3)^T$

b) $\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, $\mathbf{b} = (4,-1,0,1)^T$

11) Find the least squares line of best fit (regression line) for the data points.

a) $(-1,1),(1,0),(3,-3)$

b) $(-2,-1),(-1,0),(1,0),(2,2)$